Quiz 2 (10pts)
Math 214 Section Q1 Winter 2010

Your name:________________________________ ID#:____________

Please, use the reverse side if needed.

1.(5 pts) Test the series for convergence or divergence

$$\sum_{n=1}^{\infty} \frac{n!}{3^n(n+3)}.$$

Solution.

We will use the Ratio Test.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)!}{3^{n+1}(n+4)} \frac{3^n(n+3)}{n!} = \lim_{n\to\infty} \frac{n!(n+1)}{3^n 3(n+4)} \frac{3^n(n+3)}{n!}$$

$$= \lim_{n\to\infty} \frac{(n+1)(n+3)}{3(n+4)} = \infty$$

By the Ratio Test, the series is divergent.

2.(5 pts) Test the series for convergence or divergence

$$\sum_{n=1}^{\infty} \frac{n^2+5n}{2n^4+3n-1}.$$

Solution.

We will use the Limit Comparison Test and compare $a_n = \frac{n^2+5n}{2n^4+3n-1}$ to $b_n = \frac{1}{n^2}$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2+5n}{2n^4+3n-1} \cdot n^2 = \lim_{n\to\infty} \frac{1+\frac{5}{n}}{2+\frac{3}{n^3}-\frac{1}{n^4}} = \frac{1}{2}.$$

The latter is a nonzero constant. By the Limit Comparison Test the original series converges if and only if $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. But the latter is convergent as a p-series with $p = 2 > 1$.

Therefore, the original series is convergent.